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MODELING OF THE DEVICE FOR MEASURING THERMOPHYSICAL CHARACTERISTICS OF LIQUID BY MEANS OF LAMINAR MODE

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Symbols

a - diffusivity, m ² /s;	<i>t</i> - time, s;
$c\rho$ - volumetric heat capacity, J/(m ³ K);	T^* - the average temperature in the resistance thermometer
d - inside diameter of the tube, m;	layer, °C;
<i>hr</i> - <i>r</i> axial grid pitch;	U - temperature point of liquid, °C;
<i>hz</i> - <i>z</i> axial grid pitch;	$U_{\rm m}$ - mean temperature of liquid at the end of the tube, °C;
$\ell\mu$ - the initial fluid dynamic section length, m;	W - the inside source of heat power density;
L, Zmax - the initial and the heat-exchange sections length, m;	z - axial coordinate, m;
<i>Pe</i> - Peclet number;	$\lambda_{\rm A}$ - the liquid thermal conduction, W/(m K);
<i>Re</i> - Reynolds number;	λ - thermal conduction, W/(m·K);
<i>r</i> - radial coordinate, m;	τ - <i>t</i> time grid pitch;
<i>R</i> - inside radius of the tube, m;	$\overline{\omega}$ - liquid flow average speed, m/s;
T_{1} , T_{2} , T_{3} , T_{4} - temperature points measured in the resistance	ω_z - z axial longitudinal liquid flow speed, m/s.
thermometer layer, °C.	² ² ^{uniur rongituatiur inquita now spood, into.}

Introduction

Traditional stationary and non-stationary methods of thermophysical characteristics (TC) of liquid measuring are based on the assumption that the liquids under experiment are "quasi-hard" during the test (there is no convection heat transport in the pattern). The assumption hampers the application of these methods for measuring TC in the technological liquid flows [1, 2].

There is some information about certain methods of measuring liquid TC in laminar flow in literature. Such methods are called the means of laminar mode.

The article contains the sequential results of the research previously performed by the "Automated Systems and Devices" Department, TSTU. The mathematical model and the computer program for measuring the temperature field of the laminated axisymmetric system with the laminar liquid flow were designed in order to increase the accuracy. The mathematical model is meant to solve the direct boundary thermal conduction problem for the measuring device checking the TC of liquid by means of laminar mode. The direct boundary problem solution can be used as the basis for the numeral solution of the inverse boundary thermal conduction problem. At the stage of construction designing the program helps to choose the proper device pattern, materials and size of details of the measuring device. It contributes to forecasting the dependence of the resistance thermometer output signal of the input data, including thermophysical characteristics of the liquid (diffusivity, thermal conduction, specific heat capacity).

1. Physical Model of the Measuring Device

The measuring device is a laminated system (F. 1) that includes the tube 1, where the liquid A runs; electrical insulation 2, layer 3 with a heater, layer 4 with the copper resistance thermometers measuring T_1 , T_2 , T_3 , T_4 (at each of four sections correspondingly); separating partitions 5 between the sections; thermal insulation layer 6, decreasing the heat loss; metal cover 7 preventing the heat-transfer agent getting into the thermal insulation layer; thermal insulation plugs 8, 9; the mean temperature of liquid measuring devices 10, 11 at the entry and at the end of the tube correspondingly (T_0 and T_m temperature).

The transformer is set vertically to decrease the influence of convection on the results of the test; the liquid is delivered bottom-up. The transformer is being washed from outside with liquid *B*, which has the permanent temperature equal to that at the entry of the measuring device (*T*₀). The length of the straight part of the tube below the measuring area should be sufficient to set the speed contour at the flow sectional view close to parabolic, i.e. according to [1]: $\ell_{in}/d > 0.065 \cdot Re$, ℓ_{in} - the initial fluid dynamic section length, m; *d* - inside diameter of the tube, m; Re -Reynolds number for the liquid flow. In this case $\ell_{in} > \text{Re} \cdot 2, 6 \cdot 10^{-4}$, Re = 2300: $\ell_{in} > 0, 6 \, \text{M}$. The measuring area is divided into four sections to increase the reliability of the device. There is an extra temperature-level channel in case of failure of one of the thermometers. Besides in some variants of the calculation, while solving the inverse boundary problem, the temperature characteristics from different sections of the measuring device are used.

2. Mathematical Model of the Measuring Device

The mathematical model of the device is a system of non-stationary energy equations represented in cylindrical co-ordinates with the given initial and boundary conditions.

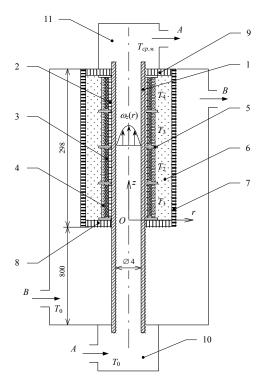


Fig. 1. Physical Model of the Measuring Device

According to [1] energy equation for each layer is as follows:

$$c\rho \left[\frac{\partial U(r,z,t)}{\partial t} + \omega_z(r)\frac{\partial U(r,z,t)}{\partial z}\right] = \frac{1}{r}\frac{\partial}{\partial r} \left[r\lambda \frac{\partial U(r,z,t)}{\partial r}\right] + \frac{\partial}{\partial z} \left[\lambda \frac{\partial U(r,z,t)}{\partial z}\right] + W(t)$$

$$r_1(z) \le r \le r_2(z), z_1(r) \le z \le z_2(r), \qquad t \ge 0$$

where: r, z - axial and radial coordinates, m; t - time, s; U - temperature, K; λ - thermal conduction, W/(m·K); $c\rho$ - volumetric heat capacity, J/(m³·K); ω_z - longitudinal liquid flow speed (different from zero for liquid and equal to zero in other layers), m/s; W - the inside source of heat power density (different from zero for the heating layer), W/m³. The initial condition for all the layers of the device is

$$U(r,z,0) = T_0$$
The boundary conditions for the liquid:
if $\mathbf{r} = 0$ (symmetry condition)
 $\partial U(0,z,t)/\partial r = 0$;
if $\mathbf{r} = \mathbf{R} (\mathbf{BC} - 4)$
 $U(R-0,z,t) = U(R+0,z,t), \ \lambda_A \frac{\partial U(R-0,z,t)}{\partial r} = \lambda_1 \frac{\partial U(R+0,z,t)}{\partial r};$
if $z = 0$ and $\mathbf{r} \le R (\mathbf{BC} - 1)$
 $U(r,0,t) = T_0.$

If z = Z, Z is the length, which is much longer than the measuring part of the tube, the test condition is added, where there is no heat flow in z direction quite apart from the measuring area. On this condition we can apply the implicit sweep method while solving the direct boundary problem

$$\partial U(r, Z, t) / \partial z = 0$$

<u>The boundary conditions for other layers of the device</u>: The boundary conditions between the layers parallel to the flow axis if $r = r_i$ (BC - 4, [1])

$$U(r_{i-0},z,t) = U(r_{i+0},z,t), \ \lambda_{i-0} \frac{\partial U(r_{i-0},z,t)}{\partial r} - \lambda_{i+0} \frac{\partial U(r_{i+0},z,t)}{\partial r} = P(t),$$

where P is the surface heat flow output at the boundaries between the layers, W/m^3 . P is usually equal to zero at all the boundaries between the layers of the device. However as an alternative we can consider the variant where the heating layer 3 (Fig. 1) is equal to zero and the heat flow escapes between the electrical insulation layer 2 and layer 4, where resistance thermometers are situated.

The conditions at the boundaries perpendicular to the flow axis, if $z = z_i$ (BC - 4)

$$U(r, z_{j-0}, t) = U(r, z_{j+0}, t), \ \lambda_{j-0} \frac{\partial U(r, z_{j-0}, t)}{\partial z} = \lambda_{j+0} \frac{\partial U(r, z_{j+0}, t)}{\partial z}$$

The conditions at the surface of the metal cover 7, if $r = r_{7max}$ (BC - 1)

 $U(r_{7\max},z,t)=T_0.$

The conditions at the surface of the thermal insulation plugs δ , 9 (without the heat flow):

 $\partial U(r, z_{8\min}, t)/\partial z = 0$, $\partial U(r, z_{9\max}, t)/\partial z = 0$.

The condition, when there is no heat flow along the *r* coordinate at the surface of the tube *l* outside the measuring area, if $z > z_{9max}$:

 $\partial U(r_{1\max}, z, t)/\partial r = 0$.

This condition in some way affects the adequacy of the mathematical model to the physical model and it is accepted just because the other variants of the boundary conditions here are even more inapplicable. To decrease the inaccuracy caused by this condition we do not take into account T_4 measured in the last section of the device.

The basic admissions of the mathematical model are:

1. All the characteristics of the liquid (and other layers of the device) are supposed not to depend on temperature, spatial value and time. It is important to mention the liquid dynamic viscosity (μ , Pa·s). This admission is correct only if the temperature changing range at the flow sectional view is minimal.

If Reynolds number is less than critical value ($\text{Re} < \text{Re}_{KP}$) we can assume that there is the Hagen - Poiseuille flow in the tube. In this case the flow speed longitudinal component can be calculated as follows:

 $\omega_{z}(r) = 2Q(1-r^{2}/R^{2})/(\pi R^{2}),$

where Q - is the liquid consumption, m³/s, R - is inside radius of the tube, m.

The liquid speed radial component ω_r is equal to zero.

2. It is assumed that there is no heat resistance at the boundaries between the layers.

3. The dissipative component of the liquid temperature rise is equal to zero. Its contribution to the Puasel flow temperature rise can be calculated as follows [4]:

$$\left(\frac{\partial U}{\partial t}\right)_{dissip.}(r) = \frac{\mu}{c\rho} \cdot \frac{16\overline{\omega}^2}{R^4} \cdot r^2 = \frac{\mu}{c\rho} \cdot \frac{16Q^2}{\pi^2 R^8} \cdot r^2$$

where $\overline{\omega}$ is the average liquid flow speed, m/s, $\overline{\omega} = Q/(\pi R^2)$.

The aim of the research is to investigate the thermophysical characteristics of the liquid. It is possible by solving the inverse boundary problem described in this mathematical model. The analytical method of the solution can not be used as the function describing the temperature distribution in any section of the device at any time is unknown. The inverse boundary problem can be solved by the numeral selection of the liquid thermal conduction λ and specification.

ic heat capacity $c\rho$ (or thermal diffusivity $a = \frac{\lambda}{c\rho}$) values, which fit the direct boundary problem (temperature field)

solution in such a way that it coincides with the values obtained from the experiment. In this case we mean the coincidence of the integral characteristics (the mean liquid temperature at the entry and at the end of the device, the average temperature in the resistance thermometer section) but not of the separate temperature values. Thus the initial aim is to work out the direct boundary problem solution strategy that is the temperature field calculation, which fits the previously mentioned mathematical model with the chosen λ and $c\rho$ (or *a*) values of the liquid.

3. The Method of Direct Boundary Problem Solving

The equation (1) can be solved numerically by means of alternating direction method [5]. First of all the equation should be made more suitable for the grid form transformation

$$\frac{\partial U(r,z,t)}{\partial t} + \omega_z(r)\frac{\partial U(r,z,t)}{\partial z} = a \left[\frac{1}{r}\frac{\partial U(r,z,t)}{\partial r} + \frac{\partial^2 U(r,z,t)}{\partial r^2} + \frac{\partial^2 U(r,z,t)}{\partial z^2}\right] + \frac{W(t)}{c\rho}$$
(2)

where *a* - is thermal diffusivity of the bad, m^2/s .

The grid of spatial value and time:

 $r_i = r_0 + i \cdot h_r, \ z_j = z_0 + j \cdot h_z, \ t_k = k \cdot \tau ,$

if h_r, h_z, τ - are grid pitches along the corresponding coordinates and time; *i*, *j*, *k* are integral numbers.

The alternating direction method means alternate substitution of the following equations for the initial differential equation (2)

$$\frac{U_{ijk+1} - U_{ijk}}{\tau} + \omega_{zi} \frac{U_{ijk} - U_{ij-1k}}{h_z} = \frac{a}{r_i} \frac{U_{i+1jk+1} - U_{i-1jk+1}}{2h_r} + a \frac{U_{i+1jk+1} - 2U_{ijk+1} + U_{i-1jk+1}}{h_r^2} + \frac{u_{i+1jk+1} - 2U_{ijk+1} + U_{i-1jk+1}}{h_r^2} + \frac{u_{i+1jk} - 2U_{ijk} + U_{ij-1k}}{h_z^2} + \frac{W_k}{c\rho}$$

$$\frac{U_{ijk+1} - U_{ijk}}{\tau} + \omega_{zi} \frac{U_{ijk+1} - U_{ij-1k+1}}{h_z} = \frac{a}{r_i} \frac{U_{i+1jk} - U_{i-1jk}}{2h_r} + a \frac{U_{i+1jk} - 2U_{ijk} + U_{i-1jk}}{h_r^2} + \frac{u_{i-1jk}}{h_r^2} + \frac{u_{i+1jk} - 2U_{ijk} + U_{i-1jk}}{h_r^2} + \frac{u_{i+1jk} - 2U_{ijk} + U_{ij-1k}}{h_r^2} + \frac{u_{i+1jk} +$$

Equation (3) is implicit along *r* coordinate and explicit along *z* coordinate, equation (4) is implicit along *z* coordinate and explicit along *r* coordinate. U_{ijk+1} temperature at the next time layer $t = t_{k+1}$ should be calculated from equation (3), if *k* is an even and from equation (4), if *k* is uneven. In this calculating scheme the temperature changes linearly in τ time pitch. The accuracy of the scheme rises as the time pitch lessens.

According to [6] the first derivatives along the longitudinal coordinate z can be approximated by centered difference if $Pe \le 2$, and by the upflow scheme if Pe > 2, Pe is Peclet number: $Pe = \overline{\omega} d/a$. It can be assumed that in this case Pe > 2 in the vast majority of the investigated liquids. Then the first derivatives are approximated by left differences if the flow coincides with the direction of axis and by right differences if the flow has the reverse direction. To approximate derivatives along the radial coordinate r centered differences are usually used.

Without *j* and k+l indexes equation (3) can be represented as:

$$A_{i}U_{i-1} - C_{i}U_{i} + B_{i}U_{i+1} = -F_{i}, \text{ where } i = 1, 2, ..., M-1$$
Coefficients A_i, B_i, C_i, F_i are calculated as follows:
$$(5)$$

$$A_{i} = \frac{a}{h_{r}^{2}} - \frac{a}{2h_{r}r_{i}}, \quad B_{i} = \frac{a}{h_{r}^{2}} + \frac{a}{2h_{r}r_{i}}, \quad C_{i} = \frac{1}{\tau} + \frac{2a}{h_{r}^{2}},$$

$$F_{i} = U_{i_{j-1k}} \left(\frac{a}{h_{z}^{2}} + \frac{\omega_{zi}}{h_{z}}\right) + U_{ijk} \left(\frac{1}{\tau} - \frac{2a}{h_{z}^{2}} - \frac{\omega_{zi}}{h_{z}}\right) + U_{i_{j+1k}} \frac{a}{h_{z}^{2}} + \frac{W_{k}}{c\rho}$$
(6)

Equation (5) can be supplemented with the boundary conditions:

(7)

In this mathematical model the first and the second type of conditions are used on the surface boundaries of the device. For the first type boundary condition $U(r, z, t) = T_0$ coefficients $\chi_1, \nu_1, \chi_2, \nu_2$ are as follows:

 $\chi_1 = 0, \ \nu_1 = T_0, \ \chi_2 = 0, \ \nu_2 = T_0.$

 $U_0 = \chi_1 U_1 + \nu_1$, $U_M = \chi_2 U_{M-1} + \nu_2$

For the second type boundary condition, where heat flow is equal to zero

 $(\partial U(r,z,t)/\partial r = 0)$ these coefficients are as follows:

$$\chi_1 = 1, v_1 = 0, \chi_2 = 1, v_2 = 0.$$

(5) - (7) set of equations is solved by means of the sweep method. Coefficients α_i , β_i are calculated in the direct sweep course:

$$\alpha_1 = \chi_1, \ \beta_1 = \nu_1, \ \alpha_{i+1} = \frac{B_i}{C_i - \alpha_i A_i}, \ \beta_{i+1} = \frac{A_i \beta_i + F_i}{C_i - \alpha_i A_i}, \text{ where } i = 1, 2, \dots, M-1.$$

The temperature values are received in the reverse sweep course:

$$U_M = \left(\nu_2 + \chi_2 \beta_M \right) / \left(1 - \alpha_M \chi_2 \right),$$

$$U_i = \alpha_{i+1}U_{i+1} + \beta_{i+1}$$
, where $i = M-1, M-2, ..., 1, 0$.

At the boundaries between the layers the fourth type boundary condition of a special kind: $\lambda_{i=0} \partial U(r_{i=0}, z, t)/\partial r - \lambda_{i=0} \partial U(r_{i+0}, z, t)/\partial r = P(t)$ may be transformed into the grid:

$$\lambda_{i-0} \frac{U_i - U_{i-1}}{h_{i-0}} - \lambda_{i+0} \frac{U_{i+1} - U_i}{h_{i+0}} = P_k \tag{8}$$

Thus coefficients α_{i+1} , β_{i+1} used in the sweep method are calculated at the section boundaries as follows:

$$\alpha_{i+1} = \frac{\lambda_{i+0}h_{i-0}}{\lambda_{i-0}h_{i+0}(1-\alpha_i) + \lambda_{i+0}h_{i-0}} , \ \beta_{i+1} = \frac{(\lambda_{i-0}\beta_i + P_kh_{i-0}) \cdot h_{i+0}}{\lambda_{i-0}h_{i+0}(1-\alpha_i) + \lambda_{i+0}h_{i-0}}$$

This scheme is implicit along r coordinate and explicit along z coordinate.

Equation (4) may be similarly transformed without *i* and k+1 indices:

 $A_{j}U_{j-1} - C_{j}U_{j} + B_{j}U_{j+1} = -F_{j}, \text{ where } j = 1, 2, ..., N-1.$ (9) Coefficients A_j, B_j, C_j, F_j are calculated as follows:

$$A_{j} = \frac{a}{h_{z}^{2}} + \frac{\omega_{zi}}{h_{z}}, \quad B_{j} = \frac{a}{h_{z}^{2}}, \quad C_{j} = \frac{1}{\tau} + \frac{\omega_{zi}}{h_{z}} + \frac{2a}{h_{z}^{2}},$$

$$F_{j} = U_{i-1jk} \left(\frac{a}{h_{r}^{2}} - \frac{a}{2h_{r}r_{i}}\right) + U_{ijk} \left(\frac{1}{\tau} - \frac{2a}{h_{r}^{2}}\right) + U_{i+1jk} \left(\frac{a}{h_{r}^{2}} + \frac{a}{2h_{r}r_{i}}\right) + \frac{W_{k}}{c\rho}$$
(10)

Equation (9) can be supplemented with the following boundary conditions:

 $U_0 = \chi_1 U_1 + \nu_1$, $U_N = \chi_2 U_{N-1} + \nu_2$.

(11)

Coefficients $\chi_1, \nu_1, \chi_2, \nu_2$ are chosen here similarly to equation (3).

(9) - (11) set of equations is solved by means of the sweep method. Coefficients α_j , β_j are calculated in the direct sweep course:

$$\alpha_1 = \chi_1, \ \beta_1 = \nu_1, \ \alpha_{j+1} = \frac{B_j}{C_j - \alpha_j A_j}, \ \beta_{j+1} = \frac{A_j \beta_j + F_j}{C_j - \alpha_j A_j}, \text{ where } j = 1, 2, \dots, N-1.$$

The temperature values are received in the reverse sweep course:

 $U_N = \left(\nu_2 + \chi_2 \beta_N\right) / \left(1 - \alpha_N \chi_2\right),$

$$U_{i} = \alpha_{i+1}U_{i+1} + \beta_{i+1}$$
, where $j = N-1, N-2, ..., 1, 0$.

At the boundaries between the layers there is the fourth type boundary condition: $\lambda_{j=0} \partial U(r, z_{j=0}, t) / \partial z = \lambda_{j+0} \partial U(r, z_{j+0}, t) / \partial z$

This condition may be transformed into the grid:

$$\lambda_{j-0} \frac{U_j - U_{j-1}}{h_{j-0}} = \lambda_{j+0} \frac{U_{j+1} - U_j}{h_{j+0}}$$
(12)

Thus coefficients α_{j+1} , β_{j+1} used in the sweep method are calculated at the section boundaries as follows:

$$\alpha_{j+1} = \frac{\lambda_{j+0} h_{j-0}}{\lambda_{j-0} h_{j+0} \left(1 - \alpha_{j}\right) + \lambda_{j+0} h_{j-0}} , \ \beta_{i+1} = \frac{\lambda_{j-0} \beta_{j} h_{j+0}}{\lambda_{j-0} h_{j+0} \left(1 - \alpha_{j}\right) + \lambda_{j+0} h_{j-0}} .$$

This scheme is implicit along z coordinate and explicit along r coordinate.

It should be noted that there are no temperature values of the grid junctions at the boundaries of the *Or* axial layers received from the equation (3). Correspondingly there are no temperature values of the grid junctions at the boundaries of the *Oz* axial layers received from the equation (4). The temperature values of the grid junctions should be explicitly calculated from the boundary conditions after the temperature field is calculated in the new time layer $t = t_{k+1}$.

Let us consider the conditions at the boundaries parallel to Or axis. If the first type condition is $U(r, z_{\min}, t) = T_0$, or $U(r, z_{\max}, t) = T_0$ the boundary temperature is T_0 : $U_{i0k+1} = T_0$, or $U_{iNk+1} = T_0$. If there is no heat flow at the boundaries $(\partial U(r, z_{\min}, t)/\partial r = 0, \text{ or } \partial U(r, z_{\max}, t)/\partial r = 0)$, then: $U_{i0k+1} = U_{i1k+1}$, or $U_{iNk+1} = U_{iN-1k+1}$. U_j temperature may be calculated in the fourth type boundary condition (12) at the layer boundaries parallel to Or axis

$$U_{ijk+1} = \frac{\lambda_{j-0}h_{j+0}U_{ij-1k+1} + \lambda_{j+0}h_{j-0}U_{ij+1k+1}}{\lambda_{j-0}h_{j+0} + \lambda_{j+0}h_{j-0}}$$

Now let us consider the conditions at the boundaries parallel to Oz axis. If the first type condition is $U(r_{\max}, z, t) = T_0$ the boundary temperature is T_0 . If there is no heat flow at the boundaries $(\partial U(0, z, t)/\partial z = 0)$, or $\partial U(r_{\max}, z, t)/\partial z = 0$, then: $U_{0,k+1} = U_{1,k+1}$, or $U_{Mjk+1} = U_{M-1,jk+1}$. U_i temperature may be calculated in the fourth type boundary condition (8) at the layer boundaries parallel to Oz axis

$$U_{i_{jk+1}} = \frac{P_k h_{i-0} h_{i+0} + \lambda_{i-0} h_{i+0} U_{i-1,jk+1} + \lambda_{i+0} h_{i-0} U_{i+1,jk+1}}{\lambda_{i-0} h_{i+0} + \lambda_{i+0} h_{i-0}}$$

It would be better to use a non-uniform grid for the liquid layer along *r* coordinate to increase accuracy. The liquid becomes thicker from the flow axis to the inner wall of the tube. So h_{ri} grid pitch is calculated in the following way: $h_{ri} = h_0 + koef \cdot i$, if koef < 1, i = 0, 1, ..., M-1.

Then h_r is calculated in the following way: $h_r = (h_{ri-1} + h_{ri})/2$.

The main characteristics recorded in the test are the average temperature points T_2 , T_3 and mean liquid temperature T_m at the end of the device. The direct boundary problem helps to find the temperature distribution $U = U_{ijk}$ at the grid junctions if $t = t_k$. These data may be used to calculate similar average temperature.

In the laminar flow inside the tube (Hagen - Poiseuille flow) the mean liquid temperature in z = Z profile is calculated in the following way:

$$U_{cp.M.}(t) = \frac{4}{R^2} \int_{0}^{R} U(r,t) \cdot r \cdot (1 - r^2/R^2) dr$$

This formula may be approximated in the grid by means of the substitution of the sum of areas under the curves described by the second power polynomial for the integral. These curves may be build according to three points: r_{i-2} , r_{i-1} , r_i for any i = 2,4,6,...,N inside the liquid layer. Then the interpolated polynomials are integrated at (r_{i-2}, r_i) area and the area sum under each of the curves. This numerical calculation of the integral is similar to Simpson's method, but it is noticed here that the grid is nonuniform along r coordinate.

 T_1 , T_2 , T_3 , T_4 temperatures in the resistance thermometer layer may be calculated in this temperature field U(r,z,t) if the temperatures are supposed to change in the middle of the layer and $r = (r_{\min} + r_{\max})/2$, where r_{\min} and r_{\max} are minimal and maximal r points in this layer. Then the average temperature is calculated in the following way:

$$T_x^*(t) = \frac{1}{z_{\max} - z_{\min}} \int_{z_{\min}}^{z_{\max}} U(z,t) dz \approx \frac{1}{N} \left(\frac{U_{j_{\min}}}{2} + \sum_{n=1}^{N-1} U_n + \frac{U_{j_{\max}}}{2} \right)$$
(trapezium method).

This mathematical model describes the temperature field inside the measuring device at the random moment $t = t_k$. But in fact the points of time of the experimental data are not properly correlated with the points of time when the temperature field is designed in the computer program. The way out is to investigate the heat exchange process if $t = \infty$ (Fig. 2, 3). The criterion proving the stationary process obtaining should be introduced in the calculating model. This criterion is the maximum relative temperature departure at the new time layer from the corresponding data at the previous time layer

$$\left\{ \left| \left(U_{\mathbf{i}_{jk+1}} - U_{ijk} \right) \middle| U_{ijk} \right| \right\}_{\max} < \varepsilon_U \, .$$

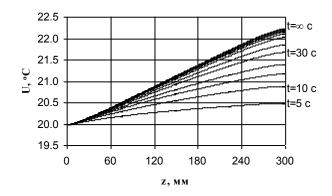


Fig. 2. Mean liquid temperature along the measuring area

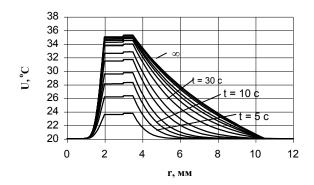


Fig. 3. Temperature distribution along the radial coordinate in the middle of the third section

Correspondingly the measured temperature points may be used in the calculation after they reach some steady point. But the approximation to the experimental data is possible to try earlier than the measured temperature points become permanent because of the iterative character of the method used in the inverse boundary problem solving.

The peculiarity of this mathematical model is that not only the heat distribution inside the liquid but also in other parts of the device is considered. Besides the heat distribution is estimated in the longitudinal (parallel to the flow axis) as well as in the radial (perpendicular to the flow axis) direction, so the solution is represented in the twodimensional axes *r*-*z*. The relative position and the size of the device items may differ but the converter characteristics should be unaffected by φ coordinate and the flow speed profile along the measuring device should be permanent.

4. The Results of the numerical simulation

There is a computer program designed according to this mathematical model. It helps to create the geometric configuration of the device (the layers relative position and the size), to represent the initial and the boundary conditions, characteristics of the layers, the time pitch and the scheme of the calculation (local one-dimensional implicit scheme or symmetric scheme of the alternating directions (Peasman-Reckford scheme)). In addition all the characteristics except the geometric configuration may be changed in the calculating process. The obtained results are represented at the screen or saved in the database.

The program considers the radial and the longitudinal directions of the heat distribution, so the solution is represented in the two-dimensional axes r-z. In the calculation all the thermophysical characteristics of the measuring device items are introduced as the temperature functions and as r and z coordinates if necessary. The device configuration may be of any type but the axisymmetric feature and the flow speed profile along the measuring device are to be permanent.

Two temperature points are written each time. The first is the average temperature T_3 along the heater area measured by means of the resistance thermometer 4 in the third section (Fig. 1). The second is the mean temperature T_m of the liquid at the end of the device (Fig. 1). The similar temperature characteristics of the temperature field inside the measuring devise are possible to calculate. The inverse boundary thermal conduction problem may be solved by means of matching the thermal conduction λ^* and the thermal diffusivity a^* indices which can satisfy the condition that the calculated temperature points T_3 and T_m are equal to the measured temperature points T_3 and T_m . The algorithm of matching is based on the progressive approximation method, which is generalized in case of two variables and non-stationary process. The obtained indexes $\lambda = \lambda^*$ and $a = a^*$ are the solutions of the inverse boundary thermal conduction problem, i.e. the required indexes of the liquid thermophysical characteristics.

This algorithm was used many times with different operating conditions of the device to solve the direct and the inverse problems. There were no signs of variability or inconsistency of the results revealed in the calculation process.

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